Mathematical Derviations

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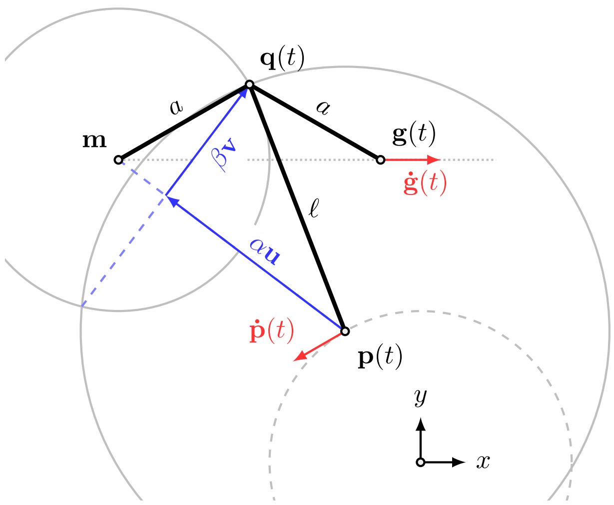

Figure 1: Sketch of the geometry problem.

1 Vibrations

The geometrical problem is easiest to solve when put in the following form, by first solving it for $\mathbf{q}(t)$:

$$(\mathbf{q}-\mathbf{p})^2 = \ell^2, \tag{1}$$
$$(\mathbf{q}-\mathbf{m})^2 = a^2. \tag{2}$$

The first step is to let

$$\mathbf{q} = \mathbf{p} + \alpha\mathbf{u} + \beta\mathbf{v}, \tag{3}$$

where $\alpha, \beta \in \mathbb{R}$ and $\mathbf{u}, \mathbf{v}$ are defined to be orthogonal to each other in the following manner:

$$\mathbf{u} = \mathbf{m} - \mathbf{p}, \qquad \mathbf{v} = \mathbf{u}^{\perp} = (u_2, -u_1)^t.$$

Then, (3) is substituted into (1) resulting in

$$\begin{aligned}\ell^2 &= (\cancel{\mathbf{p}} + \alpha\mathbf{u} + \beta\mathbf{v} - \cancel{\mathbf{p}})^2\\ &= (\alpha\mathbf{u} + \beta\mathbf{v})^2\\ &= (\alpha^2 u^2 + 2\alpha\beta\mathbf{u}\cdot\mathbf{v} + \beta^2 v^2)\end{aligned}$$

but since $\mathbf{u}$ and $\mathbf{v}$ are orthogonal to each other, and $\|\mathbf{u}^2\| = \|\mathbf{v}\|$ this expression simplifies further to

$$\ell^2 = (\alpha^2 + \beta^2)u^2.$$

Similarly, by substituting (3) in (2):

$$a^2 = \left[(1-\alpha)^2 + \beta^2\right] u^2.$$

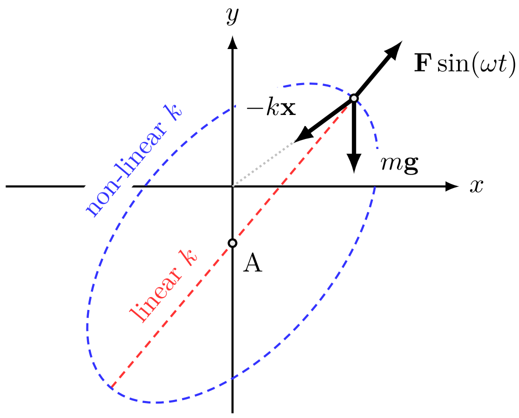

Figure 2: Vibration states.

2 Vibration dampers

Blabla introduction, Newton said that

$$m\ddot{\mathbf{x}} = \mathbf{F}\sin(\omega t) + m\mathbf{g} - k\mathbf{x}. \tag{4}$$

blabla ansatz in (4) gives